

Decoherence and damping in ideal gases

JANOS POLONYI

Strasbourg University, High Energy Theory Group, CNRS-IPHC, 23 rue du Loess, BP28 67037 Strasbourg Cedex 2 France

PACS 71.10.Ca – First pacs description
PACS 03.65.Ta – Second pacs description

Abstract. - The particle and current densities are shown to display damping and undergo decoherence in ideal quantum gases. The damping is read off from the equations of motion reminiscent of the Navier-Stokes equations and shows some formal similarity with Landau damping. The decoherence leads to consistent density and current histories with characteristic length and time scales given by the ideal gas.

Introduction. – Realistic, complex systems usually display separate length scales. In fact, a shorter length scale characterizes the elementary excitations, e.g. the average separation of particles. The interactions generates another, longer length scale, such as the mean free path. These scales are separated by some power of the coupling constants and the measurements, carried out by macroscopic devices, uncover the manifestations of the longest length scale, c.f. collective phenomena.

One usually thinks of the shortest, microscopic scale as the reflection of the dynamics of some trivial, non-interacting particle system. It is shown in this letter that contrary to this view the observables used to diagnose the system may create strong, order one effective interaction vertices for undistinguishable, noninteracting particles. What happens is that the correlations generated by the exchange statistics of the these particles produce involved correlations for local observables and create the illusion of some interactions.

The case of a quantum ideal gas is considered in this work where the effective dynamics of the particle and current densities is sought in the absence of any interaction. It is well known that for non-interacting particles these operators have one-loop, connected Green functions in arbitrary high order in contrast to the elementary fields whose higher order Green functions factorize according to Wick's theorem. The non-factorization of the Green functions indicates entanglement and suggests that the effective dynamics of the particle and the current density as local degrees of freedom might be highly nontrivial. The main result of this work is that decoherence and damping arise in the effective dynamics of the particle and current den-

sity, inherent in an ideal quantum gas.

Model and method. – Let us cope first with the issue of interactions in a free system, described by a coordinate x and the action $S[x]$. Suppose that we are interested in the dynamics of a collective coordinate $y = F(x)$. It is easy to see that the extended action

$$S[x] \rightarrow S[x, y] = S[x] - K \int dt [y(t) - F(x(t))]^2 \quad (1)$$

with K being a constant generates the correct dynamics of both coordinates. Note that the second term in the right hand side contains higher than quadratic powers of x if the collective coordinate y is a non-linear function of x . The action $S[x, y]$ offers another advantage, it allows us to consider the free coordinate x as the environment of the collective coordinate y .

We shall consider in this work a model of non-interacting particles correlated only by quantum statistics and investigate the dynamics of the particle density and current density, constituting a conserved 4-vector $j^\mu = (n, \mathbf{j})$, viewed as collective coordinates (subsystem) related to the particle degrees of freedom (environment) in a non-linear way, by means of the open time path (OTP) formalism. This is a slightly extended version of Schwinger's close time path (CTP) method [1] which provides the required general framework for exploring the open subsystem dynamics. By opening the closed time path at the final time we gain access to the density matrix.

The CTP formalism has already been used to establish decoherence and the consistency of histories [2] and to derive the transport equations [3]. While these approaches

aim at the collective mode dynamics arising from genuine interactions our goal is the construction of the effective dynamics of composite operators, particle density and current, in ideal gases. A further simplification is that the 1PI formalism is used below when the experimentally privileged local observables, the particle density and current operators are considered as the first few terms contributing to the McLaurin series of the two point functions in the Fourier space. We can make in this manner a short cut and consider the effective dynamics of the local density and current operators instead of treating the whole complexity of the two-point functions in the 2PI formalism. The result is that we do not have to follow the traditional, phenomenologically motivated way of obtaining the hydrodynamical equations from the energy-momentum conservation but can approach the problem in a more natural manner by considering directly the variational equations of the effective action for the density and current.

The path integral representation of the CTP generator functional for the connected Green functions of the 4-current density $j^\mu = \psi^\dagger C_x^\mu \psi$ [4] reads

$$e^{\frac{i}{\hbar}W[\hat{a}]} = \int D[\hat{\psi}] D[\hat{\psi}^\dagger] e^{\frac{i}{\hbar} \sum_{\sigma\sigma'} \hat{\psi}^{\dagger\sigma} (\hat{G}^{-1\sigma\sigma'} + \delta^{\sigma\sigma'} a^\sigma C) \hat{\psi}^{\sigma'}}, \quad (2)$$

where $\hat{\psi}$ and \hat{a} are CTP doublets and \hat{G}^{-1} denotes the inverse CTP propagator. We use the condensed notation where the functions defined in space-time and operators acting on them are considered as vectors and matrices, respectively. The integration over the fields $\hat{\psi}$ and $\hat{\psi}^\dagger$ yields the non-polynomial influence functional

$$W[\hat{a}] = i\xi\hbar \text{Tr} \ln(\hat{G}^{-1} + \hat{a}C). \quad (3)$$

The resulting connected Green functions with an arbitrary large number of external legs correspond to the one-loop structures build up by a single particle line, mentioned in the Introduction. We assume that the fluctuations of the current are small and retain the quadratic term

$$W^{(2)}[\hat{a}] = \hat{J}_{\text{gr}} \hat{a} - \frac{1}{2} \hat{a} \tilde{G} \hat{a}, \quad (4)$$

where $J_{\text{gr}}^{\sigma\mu} = i\xi\hbar \text{Tr}[\hat{G} C_x^{\sigma\mu}] = (\sigma n_0, \mathbf{0})$, n_0 being the particle density in the rest frame. The particle-hole propagator

$$\tilde{G}_{xx'}^{(\sigma\mu)(\sigma'\mu')} = i\xi\hbar \text{Tr}[\hat{G}^{\sigma'\sigma} C_x^\mu \hat{G}^{\sigma\sigma'} C_{x'}^{\mu'}]. \quad (5)$$

contains the exchange statistics factor ξ . Exploiting translational and rotational invariance one finally ends up with the CTP structure

$$\tilde{G} = \begin{pmatrix} L & iS^- \\ -iS^- & -L \end{pmatrix} - iS^+ \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}. \quad (6)$$

where each block $X = L, S$ is a tensor

$$X_{\omega, \mathbf{q}}^{\mu\nu} = \begin{pmatrix} X_{\omega, \mathbf{q}}^{tt} & \mathbf{q} X_{\omega, \mathbf{q}}^{ts} \\ \mathbf{q} X_{\omega, \mathbf{q}}^{st} & L X_{\omega, \mathbf{q}}^L + T X_{\omega, \mathbf{q}}^T \end{pmatrix}, \quad (7)$$

where $L = \nabla \otimes \nabla / \Delta$, $T = 1 - L$,

$$L_q^{\mu\nu} = n_s \hbar P \int_{\mathbf{k}} n_{\mathbf{k}} \left[\frac{F_{\mathbf{k}, \mathbf{q}}^{\mu\nu+}}{\omega - \frac{\hbar q^2}{2m} - \frac{\hbar \mathbf{k} \mathbf{q}}{m}} - \frac{F_{\mathbf{k}, \mathbf{q}}^{\mu\nu-}}{\omega + \frac{\hbar q^2}{2m} + \frac{\hbar \mathbf{k} \mathbf{q}}{m}} \right], \quad (8)$$

with $\int_{\mathbf{k}} = \int d^3k / (2\pi)^3$,

$$F_{\mathbf{k}, \mathbf{q}}^{\pm\mu\nu} = \begin{pmatrix} 1 & \mp \frac{\hbar}{m} (\mathbf{k} + \frac{\mathbf{q}}{2}) \\ \mp \frac{\hbar}{m} (\mathbf{k} + \frac{\mathbf{q}}{2}) & \frac{\hbar^2}{m^2} (\mathbf{k} + \frac{\mathbf{q}}{2}) \otimes (\mathbf{k} + \frac{\mathbf{q}}{2}) \end{pmatrix} \quad (9)$$

is the CTP generalization of the Lindhart function, $S^\pm = R^+ \pm R^-$,

$$R_{\omega, \mathbf{q}}^{\pm\mu\nu} = n_s \pi \hbar \int_{\mathbf{k}} \delta(\pm\omega - \omega_{\mathbf{k}+\mathbf{q}} + \omega_{\mathbf{k}}) n_{\mathbf{k}} (1 + \xi n_{\mathbf{k}+\mathbf{q}}) F_{\mathbf{k}, \mathbf{q}}^{\mu\nu\pm}, \quad (10)$$

$n_{\mathbf{k}} = 1/(e^{\beta\omega_{\mathbf{k}}} - \xi)$, $n_s = (2s+1)$ [5]. The retarded propagator is given by $G^r = -L + iS^-$.

Bare dynamics. – The bare action $S^B[\hat{J}]$ for the current expectation value $\hat{J} = \delta W[\hat{a}]/\delta \hat{a}$, defined by

$$e^{\frac{i}{\hbar}W[\hat{a}]} = \int D[\hat{J}] e^{\frac{i}{\hbar}S^B[\hat{J}] + \frac{i}{\hbar}\hat{J}\hat{a}}, \quad (11)$$

reads in the quadratic approximation

$$S^B[\hat{J}] = \frac{1}{2} (\hat{J} - \hat{J}_{\text{gr}}) \tilde{G}^{-1} (\hat{J} - \hat{J}_{\text{gr}}). \quad (12)$$

The OTP formalism is realized by sending the final time of the underlying CTP scheme to infinity and considering this action which contains no implicit boundary conditions at the final time anymore for arbitrary, not necessarily closed pairs of trajectories (J^+, J^-) . Parametrizing the source as $a^\pm = a(1 \pm \kappa)/2 \pm \bar{a}$ [4] and expressing \hat{J} by its CTP components which couple to a (physical current) and \bar{a} (auxiliary current) the current-source coupling $\hat{a}\hat{J} = aJ + \bar{a}J^a$ gives $J = (J^+ + J^-)/2 + \kappa(J^+ - J^-)/2$, $J^a = J^+ - J^-$. The inversion of the last two equations yields the form of the original current CTP components $J^\pm = J - (\kappa \mp 1)J^a/2$ in the bare action in terms of the physical and the auxiliary fields J^μ and $J^{a\mu}$. The calculation of \tilde{G}^{-1} of the bare action is tedious but straightforward and one finally obtains $S^B = \Re S^B + i\Im S^B$,

$$\begin{aligned} \Re S^B &= \frac{1}{2} \int_{\omega, \mathbf{k}} \left[\frac{L_{\omega, \mathbf{k}}^{tt} n_{\omega, \mathbf{k}}^* n_{\omega, \mathbf{k}}^a - i S_{\omega, \mathbf{k}}^{tt-} n_{\omega, \mathbf{k}}^* n_{\omega, \mathbf{k}}^a}{(1+z^2)[(L_{\omega, \mathbf{k}}^{tt})^2 + (S_{\omega, \mathbf{k}}^{tt-})^2]} \right. \\ &\quad \left. + \frac{L_{\omega, \mathbf{k}}^T \mathbf{j}_{\omega, \mathbf{k}}^{T*} \mathbf{j}_{\omega, \mathbf{k}}^{Ta} - i S_{\omega, \mathbf{k}}^{T-} \mathbf{j}_{\omega, \mathbf{k}}^{T*} \mathbf{j}_{\omega, \mathbf{k}}^{Ta}}{(L_{\omega, \mathbf{k}}^T)^2 + (S_{\omega, \mathbf{k}}^{T-})^2} \right], \\ \Im S^B &= \frac{1}{4} \int_{\omega, \mathbf{k}} \left[\frac{S_{\omega, \mathbf{k}}^{tt+} n_{\omega, \mathbf{k}}^{a*} n_{\omega, \mathbf{k}}^a}{(1+z^2)[(L_{\omega, \mathbf{k}}^{tt})^2 + (S_{\omega, \mathbf{k}}^{tt-})^2]} \right. \\ &\quad \left. + \frac{S_{\omega, \mathbf{k}}^{T+} \mathbf{j}_{\omega, \mathbf{k}}^{Ta*} \mathbf{j}_{\omega, \mathbf{k}}^{Ta}}{(L_{\omega, \mathbf{k}}^T)^2 + (S_{\omega, \mathbf{k}}^{T-})^2} \right], \end{aligned} \quad (13)$$

with $z = m\omega/\hbar k_{\text{gas}} k$ where k_{gas} is the characteristic wave vector of the one-particle distribution function n_k , i.e. the Fermi wave-vector k_F , for fermions and the thermal wave-vector $\sqrt{mk_B T}/\hbar$ for bosons without condensate.

Consistency and decoherence. – The usual way of discovering decoherence is either to approximate the system-environment interactions as a successive scattering process [9, 10] or to eliminate a direct product factor of the Hilbert space in some simple model [11, 12]. The former approach offers a suggestive view of the dynamical origin of decoherence but remains rather qualitative. The latter, more formal method orients our interest towards the degeneracy as the source of decoherence. But to establish a sufficiently degenerate environment one needs a large number of degrees of freedom and such a system can be handled by means of quantum field theory only. This is the point where the OTP/CTP schemes arise in a natural manner because they can handle the (reduced) density matrix by means of Green functions in many-body systems.

$\Im S^B$, which is equal to the imaginary part of the influence functional, controls the consistency [6–8] of the pairs of OTP trajectories of the subsystem, running to the final time t_f . Furthermore, the dependence of $\Im S^B$ on t_f characterizes the building up of the decoherence [9–12] of the environment states $|e^\pm(t_f)\rangle$, generated from $|e\rangle$ by the time evolution in the presence of fixed system trajectories, $\langle e^+(t_f)|e^-(t_f)\rangle = \exp iS^B/\hbar$. The reduced density matrix of the subsystem $\langle \psi^+ | \text{Tr}_e \rho(t_f) | \psi^- \rangle = \exp iW[\hat{J}]/\hbar$ is given by Eq. (2). If the decoherence of an environment state is robust then it is referred to as pointer state [11]. A necessary condition for robustness is the persistence of the decoherence for macroscopic times, i.e a sufficiently large value of $\Im S^B$ for trajectories with $\omega \rightarrow 0$. In the present consideration the subsystem is characterized by the 4-vector $(n_{t,\mathbf{k}}, \mathbf{j}_{t,\mathbf{k}})$ for each fixed wave vector \mathbf{k} , and $\Im S^B$ indeed controls the consistency of pairs of small amplitude oscillatory trajectories.

To construct a measure for consistency and decoherence we choose a time dependent state in which the expectation value J^μ is a plane wave with amplitude $n_{\omega,\mathbf{k}}, \mathbf{j}_{\omega,\mathbf{k}}$ and consider the overlap with states with infinitesimally different amplitudes. For a single mode, the bare functional integral of Eq. (11) assumes the form of ordinary integrals leading to the expectation values $\langle n^* n \rangle = -2\hbar S^{tt+}(1+z^2)$, $\langle n^* n^a \rangle = 2i\hbar(L^{tt} + iS^{tt-})(1+z^2)$, $\langle \mathbf{j}^{T*} \mathbf{j}^T \rangle = -2\hbar S^{T+}$, $\langle \mathbf{j}^{T*} \mathbf{j}^{Ta} \rangle = 2i\hbar(L^T + iS^{T-})$ (the indices ω, \mathbf{k} being suppressed). The inverse of the second moment of J^a will be chosen as a measure of consistence or decoherence [13]. Since $\langle n^a n^a \rangle = \langle \mathbf{j}^{Ta*} \mathbf{j}^{Ta} \rangle = 0$ in the absence of external source we use instead the ratios

$$\begin{aligned} (D^{tt})^2 &= \frac{\hbar |\langle n^* n \rangle|}{|\langle n^* n^a \rangle|^2} = \frac{|S^{tt+}|}{2(1+z^2)[(L^{tt})^2 + (S^{tt-})^2]}, \\ (D^T)^2 &= \frac{\hbar |\langle \mathbf{j}^{T*} \mathbf{j}^T \rangle|}{|\langle \mathbf{j}^{T*} \mathbf{j}^{Ta} \rangle|^2} = \frac{|S^{T+}|}{2[(L^T)^2 + (S^{T-})^2]}, \end{aligned} \quad (14)$$

for this purpose. Large D indicates that the quantum fluctuations around the trajectory with given, oscillatory plane wave expectation value for J are distributed by mainly classical probabilities. They decohere and the

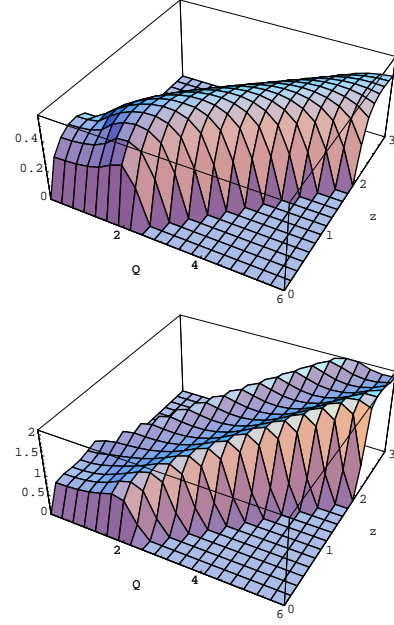


Fig. 1: D^{tt} (upper) and D^T (lower) as functions of Q and z .

neighboring trajectories tend to be consistent. One expects that classical probabilities are recovered at long distances and times. The time and distance scale of the restoration of classical probabilities can be estimated by the value of $1/\omega$ and $1/|\mathbf{k}|$ where these ratios become large. In the absence of other dimensional constants than \hbar , the particle mass m and the density n of the ideal gas these scales are in the order of magnitude of $\hbar/\epsilon_{k_{\text{gas}}}$, $1/k_{\text{gas}}$ and fall into the quantum domain. Thus consistency or decoherence occur at microscopic scale for ideal gases and the quantum-classical crossover remains inaccessible for macroscopic devices.

The ratios given by Eqs. (14) are plotted in Fig. 1. They are nonvanishing in two strips, starting from the origin of the plane (Q, z) and having slopes $\pm 1/2$ and width 2. The center of the strips is the straight line $z = \pm Q/2$, corresponding to the mass-shell condition of the environment, $\hbar\omega = \hbar^2 q^2/2m$. The value of the functions along this line is shown in Fig. 2. The most consistent density trajectories are around $(Q, z) = (2, 1)$. The classical features might be enhanced in the IR directions by means of appropriate interactions. The current trajectories become more consistent when we move towards the UV direction. This does not imply classical behavior at short distances or times, the decoherence and consistency being necessary but not sufficient conditions for classical physics. There are few particles involved at short space or time scales and the quantum fluctuations are important. Furthermore they do not allow the recovery of the deterministic classical laws. The qualitative difference of D^{tt} and D^T suggests that the scale of the quantum-classical crossover is determined by the density rather than the current fluctuations.

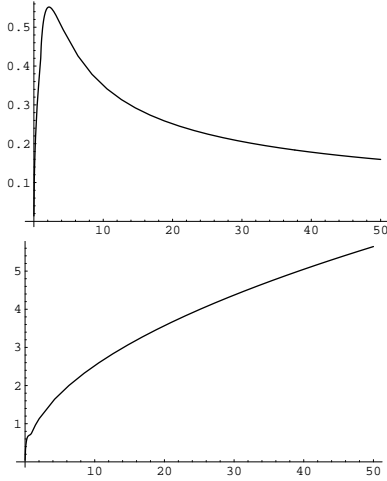


Fig. 2: D^{tt} (upper) and D^T , (lower) plotted along the line $z = Q/2$ as functions of Q .

Equations of motion. – The equation of motion for the expectation value of the 4-current expectation value $J = \delta \mathbb{R}W[a, \bar{a}]/\delta a$ is obtained from the effective action $\Gamma[J] = \mathbb{R}W[a, \bar{a}] - a \cdot J$ whose quadratic part in the fields is

$$\kappa \Gamma[J] = J \tilde{G}^{n-1} (\tilde{G}^r \bar{a} + J_{\text{gr}}) - \frac{1}{2} J \tilde{G}^{n-1} J. \quad (15)$$

Clearly, the effective action is well defined for $\kappa \neq 0$ only [4]. The corresponding equation of motion

$$\tilde{G}^{r-1} (J - J_{\text{gr}}) = \bar{a} \quad (16)$$

is diagonal in Fourier space. The highly complicated, non-polynomial equations simplify considerably in the hydrodynamical limit defined by $\tilde{k} = k/k_{\text{gas}} \ll 1$, $z \ll 1$,

$$\begin{aligned} -g^0 \bar{a}^0 &= \left(a_0 + a_z \frac{m}{\hbar k_{\text{gas}}} \frac{i\omega}{k} + \frac{a_{kk}}{k_{\text{gas}}^2} k^2 \right) n, \\ -g^0 \bar{\mathbf{a}}^L &= \left(a_0 + a_z \frac{m}{\hbar k_{\text{gas}}} \frac{i\omega}{k} + \frac{a_{kk}}{k_{\text{gas}}^2} k^2 \right) \mathbf{j}^L, \\ -g^T \bar{\mathbf{a}}^T &= \left(b_0 + b_z \frac{m}{\hbar k_{\text{gas}}} \frac{i\omega}{k} + \frac{b_{kk}}{k_{\text{gas}}^2} k^2 \right) \mathbf{j}^T, \end{aligned} \quad (17)$$

where $g^0 = n_s m k_F / 4\pi^2$, $g^T = g^0 \hbar^2 k_F^2 / m^2$. For zero temperature fermions the numerical constants are $a_0 = 1/2$, $a_z = \pi/4$, $a_{kk} = 1/24$, $b_0 = 3/2$, $b_z = 9\pi/4$ and $b_{kk} = 39/128$. These equations are reminiscent of the linearized Navier-Stokes equation of hydrodynamics. The different coefficients appearing for the longitudinal and the transverse current introduce different bulk and shear viscosity effects.

To see the nature of the dissipative terms appearing in Eqs. (17) more clearly we consider a stationary flow in the w -direction which depends on the x -coordinate only. For the static deviation of the density and the current density from the corresponding ground state expectation values

Eqs. (17) yield

$$\begin{aligned} n_{\mathbf{k}} &= g^0 \left[\left(\frac{1}{\tilde{k}} - \frac{\tilde{k}}{4} \right) \ln \left| \frac{2 - \tilde{k}}{2 + \tilde{k}} \right| - 1 \right] a_{\mathbf{k}}^0, \\ \mathbf{j}_{\mathbf{k}}^T &= g^T \left[\frac{\tilde{k}^2}{16} + \frac{1}{\tilde{k}} \left(1 - \frac{\tilde{k}^2}{4} \right)^2 \ln \left| \frac{2 - \tilde{k}}{2 + \tilde{k}} \right| - \frac{5}{12} \right] \bar{\mathbf{a}}_{\mathbf{k}}^T \end{aligned} \quad (18)$$

The longitudinal component of the current, \mathbf{j}^L , can be reconstructed from these equations by means of the continuity equation for the current. Note that Eqs. (18) contain all higher order derivatives. The only approximation involved is the linearization of the equation of motion. For the sake of simplicity (18) is approximated by

$$\begin{aligned} n_{\mathbf{k}} &\approx -2g^0 e^{-\frac{Q^2}{6}} \bar{a}_{\mathbf{k}}^0, \\ \mathbf{j}_{\mathbf{k}}^T &\approx -\mathbf{z} g^T \frac{2}{3} e^{-\frac{Q^2}{3}} \bar{\mathbf{a}}_{\mathbf{k}}^T. \end{aligned} \quad (19)$$

We choose a weak external source

$$\bar{a}_{\mathbf{x}}^X = \frac{u^X}{(2\pi \ell_{\text{ext}}^2)^{3/2}} e^{-\frac{x^2}{2\ell_{\text{ext}}^2}}, \quad (20)$$

where the index X stands for 0 or T . It yields a Gaussian flow,

$$\begin{aligned} n_x &= -2 \frac{g^0 u^0}{(2\pi \ell_{\text{flow}}^2)^{3/2}} e^{-\frac{x^2}{2\ell_{\text{flow}}^2}}, \\ \mathbf{j}_x^T &= -\mathbf{z} \frac{2}{3} \frac{g^T u^T}{(2\pi \ell_{\text{flow}}^2)^{3/2}} e^{-\frac{x^2}{2\ell_{\text{flow}}^2}}, \end{aligned} \quad (21)$$

with $\ell_{\text{flow}} = \sqrt{\ell_{\text{ext}}^2 + 1/3k_F^2}$. Such a spread of the external perturbation is a collective phenomenon due to the presence of k_F in its length scale and is reminiscent of a diffusive process.

It is easy to find the dynamical origin of this diffusion, it is the well known spread of wave-packets of a free particle. In fact, the plane wave states not only serve as a useful basis for free particles, they represent states which, one-by-one, produce robust expectation values. Any smearing of the wave function of this state in the momentum representation to an ordinary wave-packet which is a linear superposition of plane waves without some special fine tuning of the relative phases leads to the complete loss of localization as the time passes, according to the Riemann-Lebesgue lemma. The same spread of the individual plane wave components spreads any local observable such as the density or current. The external source for the three-current, \mathbf{a} , can be interpreted as a space-dependent shift of the momentum variable, eg. $p_z \rightarrow p_z + \bar{a}^T$ in case of the external source (20). The spatial spread of such a local Galilean-boost, shown in the second equation of (21), indicates the presence of shear viscosity in the dynamics of the three-current.

One finds similar processes in a collisionless plasma. The retarded solution of the collisionless Boltzmann equation in the presence of an electric field and the Maxwell

equations produces Landau damping, the spread of the energy of the electric field over the charges [14]. Though there is no collision term in the Boltzmann equation, its simultaneous solution with the equation of motion for the electric field introduces genuine interactions among the charges. The spread, described by Eqs. (21) corresponds to more elementary processes than the Landau damping because it takes place in a truly noninteracting quantum gas. A formal similarity between the two phenomena is the dephasing, taking place either in the solution of the Schrödinger equation for ideal quantum gases or in the Fourier integral representation of the retarded Lienard-Wiechert potential for classical plasma [15].

One should bear in mind the difference between the spread of the wave packet and truly diffusive processes. The former is related to the asymmetry of the initial and final states in physically motivated bases and arises in time reflection symmetric dynamics, governed by the Schrödinger equation. The nontrivial, dynamical issue of the time arrow, the dynamical breakdown of the time reversal invariance appears in the latter when irreversibility is observed. The key element is that the soft, gapless collective modes have a time scale which is much longer than the characteristic time of the observation. The bosonisation is an exact rewriting of the dynamics of a fermion system with conserved particle number and it corresponds within the functional formalism to the retaining of all Green functions of even order as independent variables when the fermion fields are integrated over in the path integral expressions. Instead of such an extremely involved dynamics our calculation can be considered as an approximative bosonisation where one keeps the density and current, the first two terms of McLaurin series of the two-point function, ignoring the remaining informations in the two-point function and all higher order Green functions. Such a coarse graining retains the long distance, low frequency part of the dynamics and the resulting effective theory applies for scales far in the infrared compared to the intrinsic scale of the ideal gas. Such a loss of information related to the strong separation of scales generates the dynamical breakdown of the time reversal invariance and irreversibility. Had we included enough informations in the effective description to extend it to the scale of the ideal gas we would have lost the dissipative terms.

Summary. — It is pointed out in this paper that the statistics of undistinguishable and noninteracting particles produces complicated, nonpolynomial interaction vertices when the dynamics is diagnosed by composite operators. In the physically motivated choice of the density and current operators one finds damping and decoherence in an ideal gas. These phenomena arise at the only microscopic scale of the ideal gas and differ from the analogous phenomena of truly interactive systems. Namely, the damping leads to irreversibility for processes far infrared compared to the intrinsic scale of the ideal gas and the decoherence of the current operator is stronger at shorter length and

time scales. These results provide a better background to interpret damping and the classical limit of interacting systems. In particular, they clarify the physical content of the UV initial conditions when the renormalization group method is applied to discover the quantum-classical transition.

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I thank the number of discussions with Janos Hajdu which gave encouragement and insight during the preparation of this work.

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